Name: _____

Higher Unit 15 topic test

Date:

Time: 65 minutes

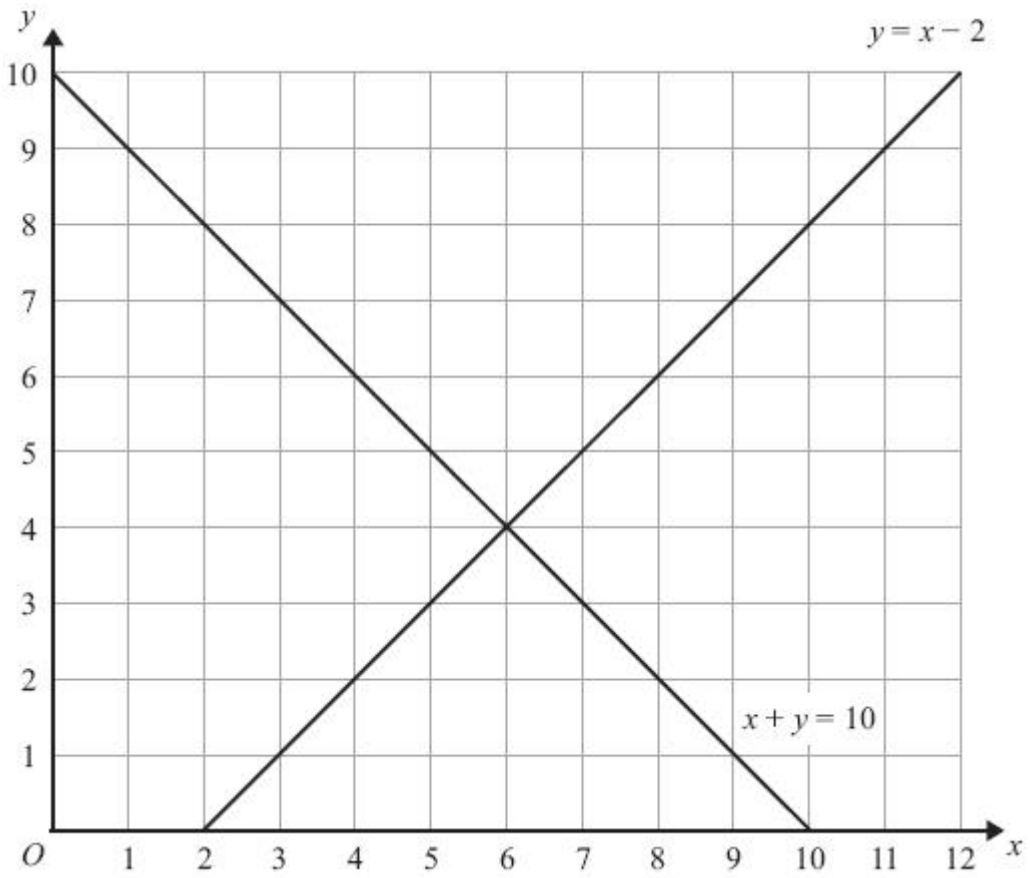
Total marks available: 59

Total marks achieved: _____

Questions

Q1.

The lines $y = x - 2$ and $x + y = 10$ are drawn on the grid.



On the grid, mark with a cross (**×**) each of the points with integer coordinates that are in the region defined by

$y > x - 2$

$x + y < 10$

$x > 3$

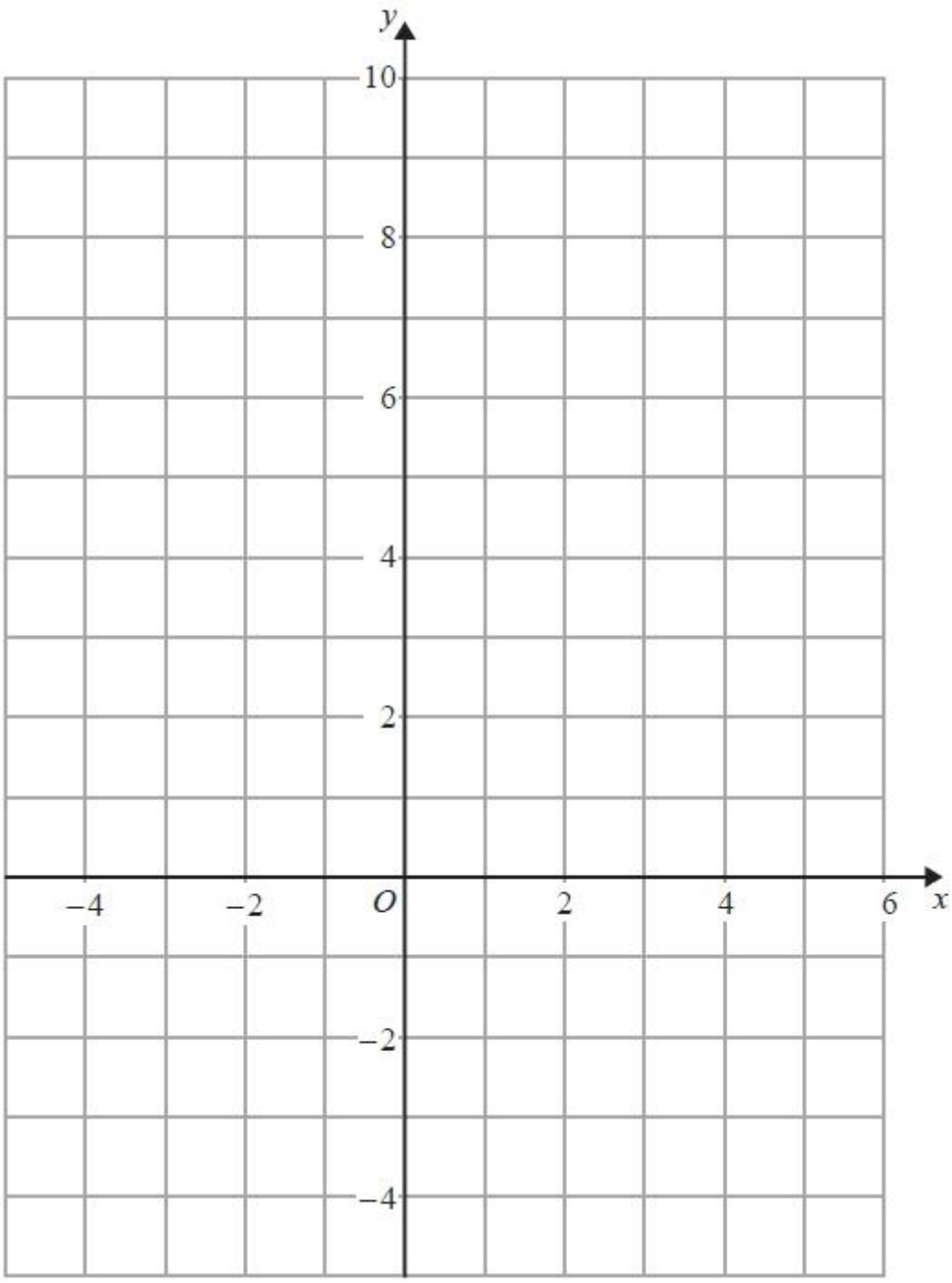
(Total for Question is 3 marks)

Q2.

On the grid, shade the region that satisfies all these inequalities.

$x + y < 4$ $y > x - 1$ $y < 3x$

Label the region **R**.



(Total for question is 4 marks)

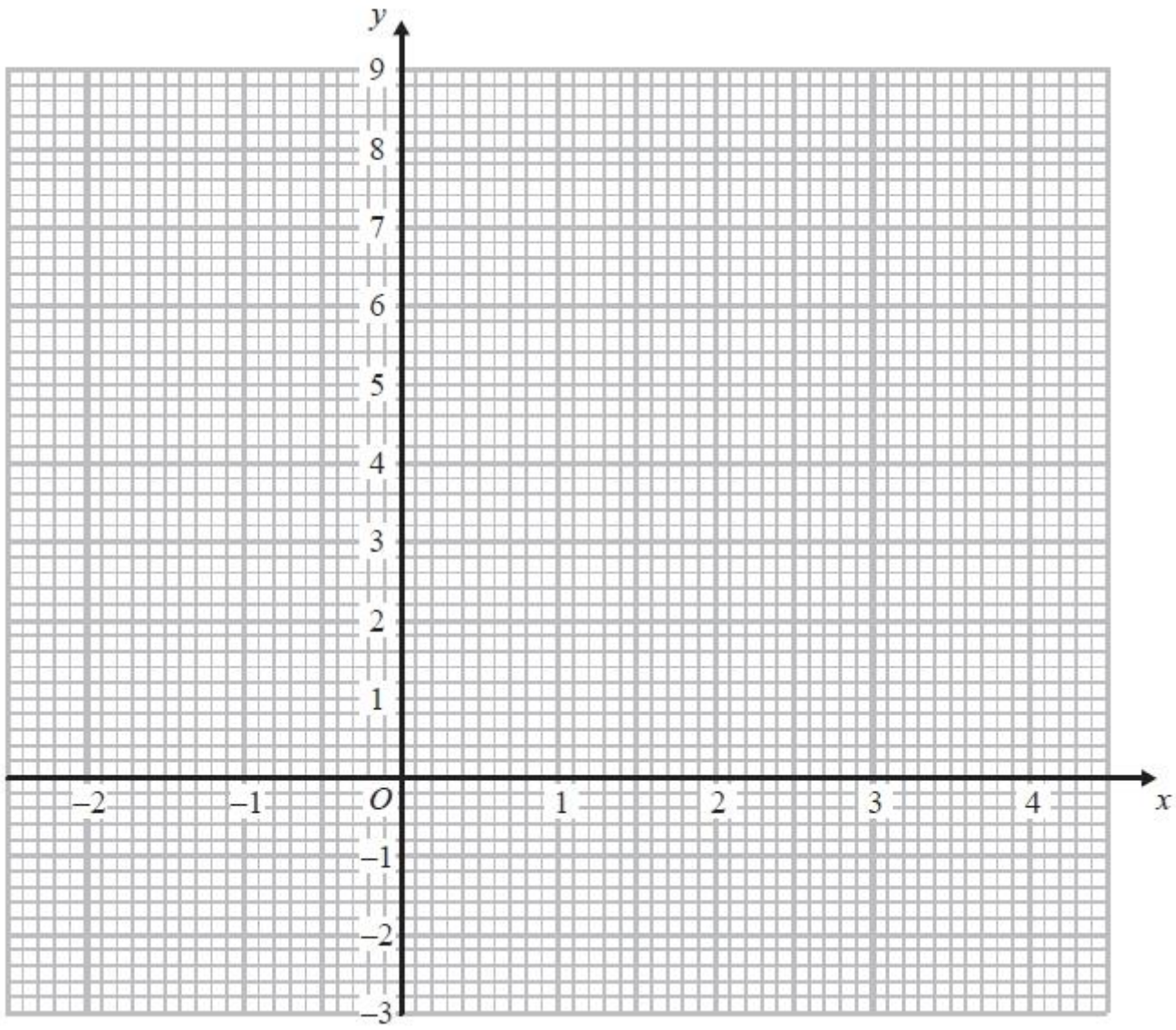
Q3.

(a) Complete the table of values for $y = x^2 - 2x - 1$

x	−2	−1	0	1	2	3	4
y	7			−2	−1		

(2)

(b) On the grid, draw the graph of $y = x^2 - 2x - 1$ for values of x from −2 to 4



(2)

(c) Solve $x^2 - 2x - 1 = x + 3$

.....

(2)

(Total for Question is 6 marks)

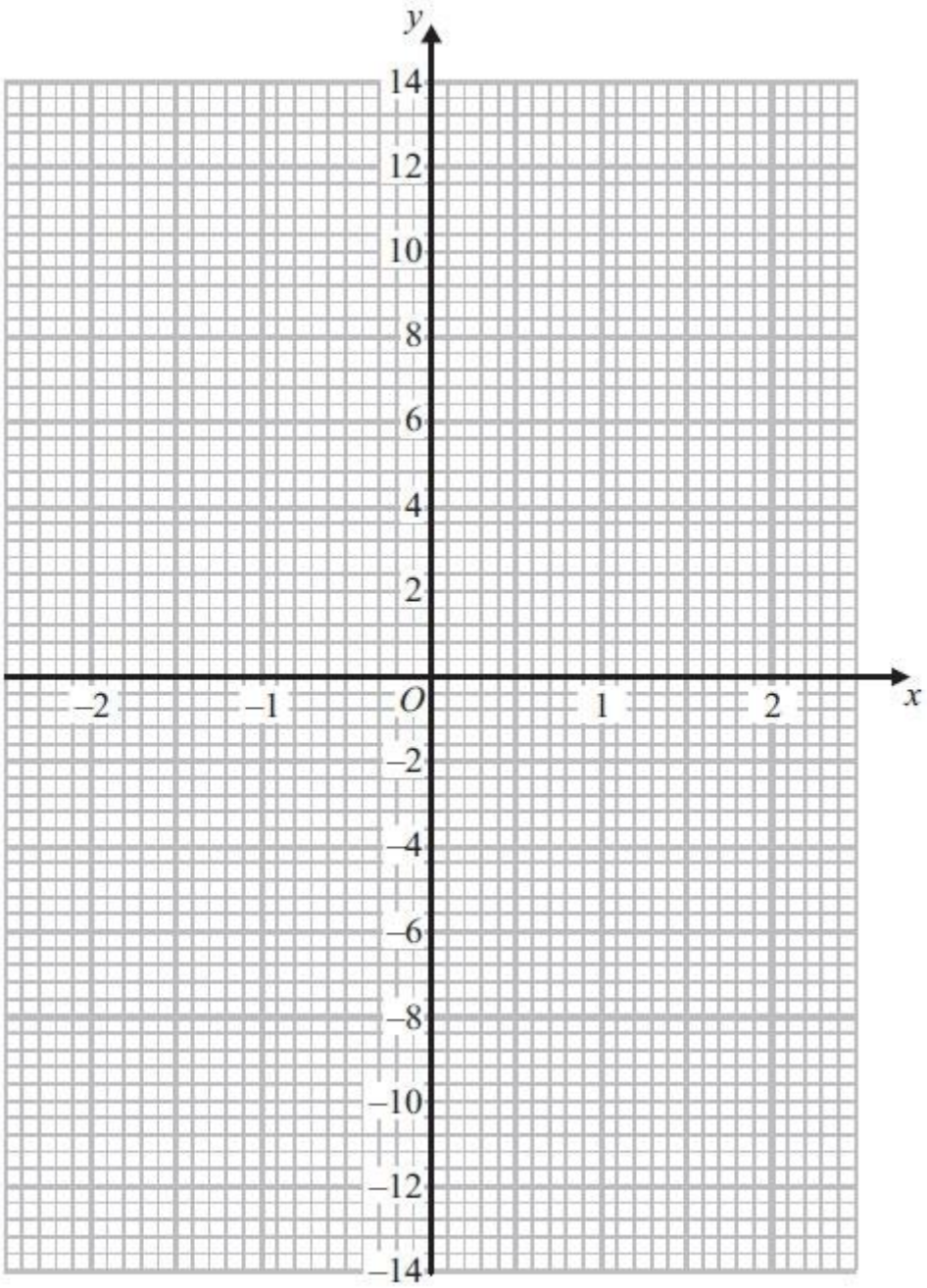
Q4.

(a) Complete this table of values for $y = x^3 + 2x - 1$

x	-2	-1	0	1	2
y		-4			11

(2)

(b) On the grid, draw the graph of $y = x^3 + 2x - 1$

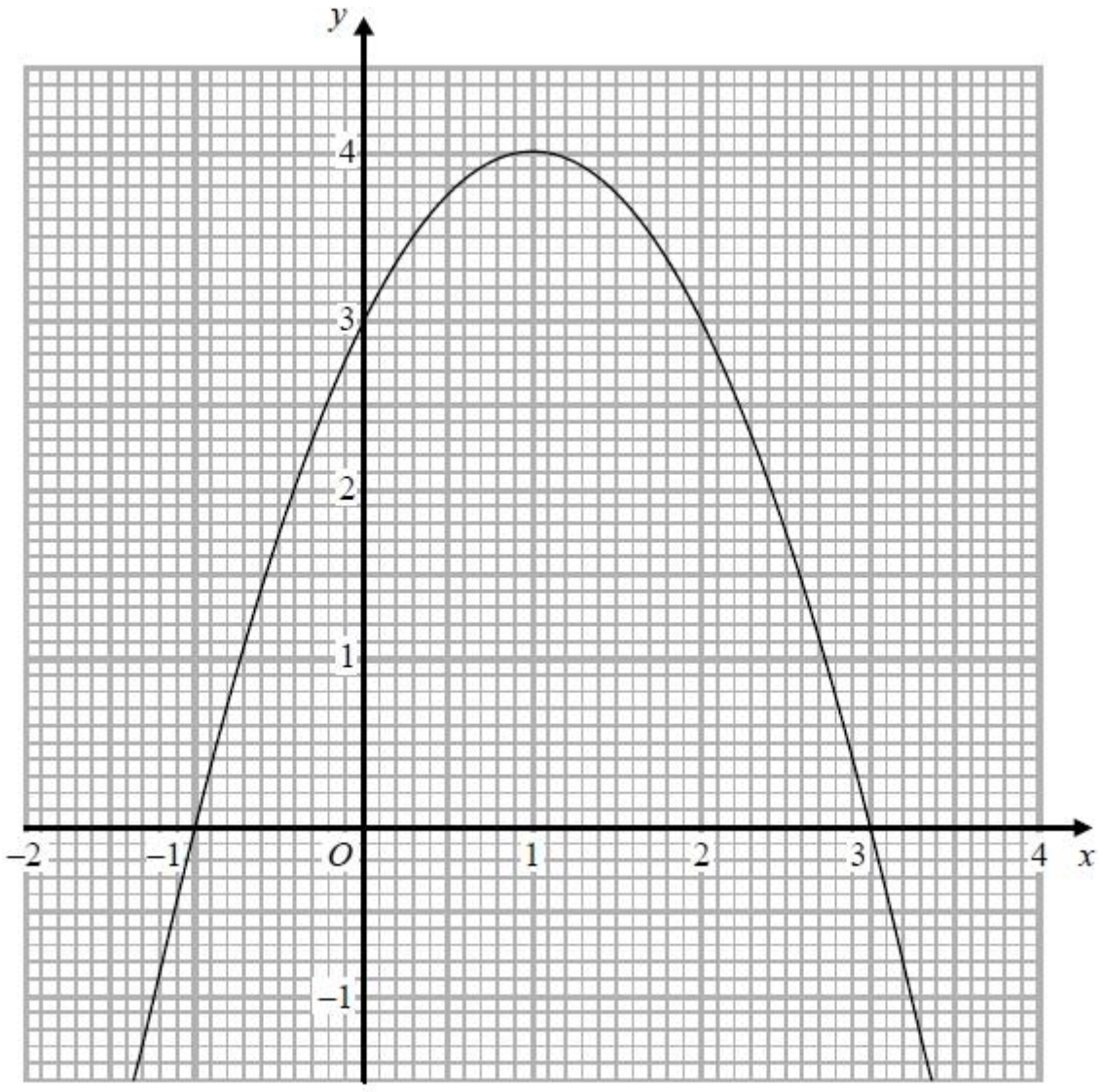


(2)

(Total for Question is 4 marks)

Q5.

The graph of $y = f(x)$ is drawn on the grid.



(a) Write down the coordinates of the turning point of the graph.

(....., ...)

(1)

(b) Write down the roots of $f(x) = 2$

.....

(1)

(c) Write down the value of f(0.5)

.....

(1)

(Total for question = 3 marks)

Q6.

The expression $x^2 - 8x + 21$ can be written in the form $(x - a)^2 + b$ for all values of x.

(a) Find the value of a and the value of b.

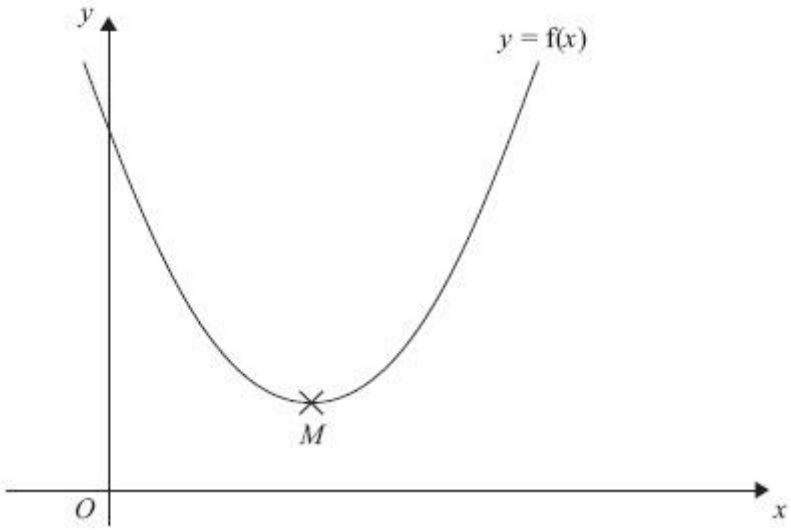
a = ...

b = ...

(3)

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 8x + 21$

The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M.

(b) Write down the coordinates of M.

.....

(1)

(Total for Question is 4 marks)

Q7.

Show that

$$(3x - 1)(x + 5)(4x - 3) = 12x^3 + 47x^2 - 62x + 15$$

for all values of x.

(Total for question is 3 marks)

Q8.

Solve $x^2 > 3x + 4$

.....

(Total for question = 3 marks)

Q9.

Solve the inequality $x^2 > 3(x + 6)$

.....

(Total for question = 4 marks)

Q10.

(i) Sketch the graph of f(x) = x^2 − 5x + 10, showing the coordinates of the turning point and the coordinates of any intercepts with the coordinate axes.

(ii) Hence, or otherwise, determine whether f(x + 2) − 3 = 0 has any real roots.
Give reasons for your answer.

(Total for question = 6 marks)

Q11.

The number of slugs in a garden t days from now is p_t where

$p_0 = 100$
$p_{t+1} = 1.06p_t$

Work out the number of slugs in the garden 3 days from now.

.....

(Total for question = 3 marks)

Q12.

(a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be arranged to give $x = \frac{1}{4} - \frac{x^3}{4}$

(1)

(c) Starting with $x^0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$ twice, to find an estimate for the solution of $x^3 + 4x = 1$

.....

(3)

(Total for question = 6 marks)

Q13.

The number of bees in a beehive at the start of year n is P_n.
The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

.....

(Total for question is 3 marks)

Q14.

(a) Show that the equation $3x^2 - x^3 + 3 = 0$ can be rearranged to give

$$x = 3 + \frac{3}{x^2}$$

(2)

(b) Using

$$x_{n+1} = 3 + \frac{3}{x_n^2} \quad \text{with } x_0 = 3.2,$$

find the values of x_1, x_2 and x_3

.....

(3)

(c) Explain what the values of x_1, x_2 and x_3 represent.

.....

.....

(1)

(Total for question is 6 marks)

Examiner's Report

Q1.

Many candidates failed to attempt this question, and of those who did, it was most common to see a plethora of crosses, usually well away from the desired region. Many ignored the line $x = 3$.

Q2.
No Examiner's Report available for this question

Q3.

The values of y corresponding to positive values of x were generally worked out correctly. There was less success with the negative values, especially the value of y at = −1. In part (b) values were generally plotted accurately and the points joined with a smooth curve, although the occasional set of straight line segments was also seen. Part (c) proved beyond most candidates. Correct solutions were split between those who connected up the whole question and drew the straight line with equation $y = x + 3$. They were then able to pick out the required values of x for the two marks. Other candidates restarted, rearranged the equation and solved it, usually by factorisation. If the two values of x were given then the marks were awarded. Some candidates spotted that $x = 4$ satisfies the original equation, but without any of the two approaches shown they did not score any marks.

Q4.

The majority of candidates gained full marks for this question, finding the missing values and drawing a correct graph. Very few candidates failed to calculate at least one correct value. The points were usually accurately plotted although the point (2, 11) was sometimes plotted at (2, 13). Some candidates only gained one mark in part (b) as they joined the points with straight lines rather than drawing the curve freehand. Some did not join the points at all and some drew a line of best fit for the points. Curves were sometimes inaccurate, not passing through the points exactly or drawn with too thick a line or with several lines. Some candidates seemed to have pre-conceived ideas as to what the graph should look like and drew a parabola that contradicted their calculations.

Q5.
No Examiner's Report available for this question

Q6.

This question was poorly answered. It was clear that only a small minority of candidates were well practised in the technique of completing the square. Candidates who realised what was required often went on to carry out this technique but then spoiled their responses by writing a = −4, b = 5. Other candidates wrote $(x + 4)^2 + 5$ then $a = 4$, $b = 5$. This was clearly incorrect working and could not be awarded the marks. "8" and "21" were commonly seen incorrect answers. Part (b) was answered correctly by only a small minority of candidates with many of the more able candidates failing to see the connection between the two parts of the question.

Q7.
No Examiner's Report available for this question

Q8.
No Examiner's Report available for this question

Q9.
No Examiner's Report available for this question

Q10.
No Examiner's Report available for this question

Q11.
No Examiner's Report available for this question

Q12.
No Examiner's Report available for this question

Q13.
No Examiner's Report available for this question

Q14.
No Examiner's Report available for this question

Mark Scheme

Q1.

Question		Working	Answer	Mark	Notes
			(4,3), (4,4), (4,5), (5.4) marked	3	M2 for identifying the correct region or at least 3 correct points with no more than 3 incorrect points (M1 for drawing $x = 3$ (solid or dashed line) or at least 1 correct point with no more than 3 incorrect points) A1 cao

Q2.

Paper 1MA1:3H			
Question	**Working**	**Answer**	**Notes**
		Region R	M1 for one line correctly drawn M1 for two or more lines correctly drawn A1 for a correct region indicated between two correct lines A1 fully correct region indicated with all lines correct

Q3.

PAPER: 1MA0_1H					
Question		**Working**	**Answer**	**Mark**	**Notes**
	(a)		2, –1, 2, 7	2	B2 for all correct (B1 for 2 or 3 correct)
	(b)		Correct graph	2	M1 (dep on at least B1) for at least 6 points from their table plotted correctly A1 cao for fully correct graph
	(c)	$x^2 - 3x - 4 = 0$ $(x - 4)(x + 1) = 0$	–1, 4	2	M1 for line $y = x + 3$ drawn correctly or for reduction to correct 3 term quadratic (=0) and : $(x \pm 1)(x \pm 4)$ or formula using $a = 1$, $b = -3$ and $c = -4$, allow one sign error in the formula, or $\left(x - \frac{3}{2}\right)^2 = 4 + \left(\frac{3}{2}\right)^2$ A1 cao

Q4.

		Working	Answer	Mark	Notes
	(a)		–13, –1, 2	2	B2 for all values correct (B1 for any one value correct)
	(b)		Graph drawn	2	M1 ft for at least 4 points plotted correctly from their table A1 cao for correct curve drawn from (–2, –13) to (2, 11)

Q5.

Paper 1MA1: 2H			
Question	Working	Answer	Notes
(a)		(1, 4)	B1
(b)		–0.4, 2.4	B1
(c)		3.75	B1 accept 3.7 – 3.8

Q6.

		Working	Answer	Mark	Notes
	(a)		$a = 4$, $b = 5$	3	M1 for sight of $(x – 4)^2$ M1 for $(x – 4)^2 – 16 + 21$ A1 for $a = 4$, $b = 5$ OR M1 for $x^2 – 2ax + a^2 + b$ M1 for $–2a = – 8$ and $a^2 + b = 21$ A1 for $a = 4$, $b = 5$
	(b)		(4, 5)	1	B1 ft

Q7.

Paper 1MA1: 2H			
Question	**Working**	**Answer**	**Notes**
		Fully correct algebra to show given result	M1 for method to find the product of any two linear expressions; eg. 3 correct terms or 4 terms ignoring signs M1 for method of 6 products, 4 of which are correct (ft their first product) A1 for fully accurate working to give the required result

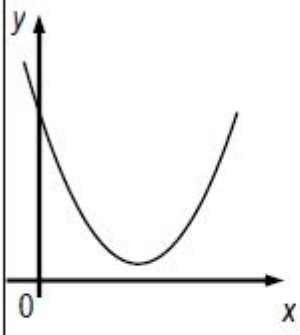
Q8.

Paper 1MA1: 1H			
Question	**Working**	**Answer**	**Notes**
		$x > 4$, $x < -1$	M1 rearrange quadratic and factorise M1 critical values of 4 and −1 found A1

Q9.

Question	Working	Answer	Notes
		$x < -3, x > 6$	M1 Rearrange to $x^2 - 3x - 18 > 0$ M1 Correct method to solve $x^2 - 3x - 18 = 0$ M1 Establish critical values −3 and 6 A1 $x < -3, x > 6$

Q10.

Question	Working	Answer	Mark	AO	Notes
(i)			M	1.3b	M1 for $(x-2.5)^2-(2.5)^2+10$ **or** attempt to find points to plot – must have at least 3 correct points evaluated
			A	1.3b	A1 for $(x-2.5)^2+3.75$ or parabola with minimum marked at (2.5, 3.75)
			C	2.3b	C1 for parabola drawn with minimum in 1st quadrant **or** y intercept at (0, 10)
			C	2.3b	C1 for parabola drawn with minimum in 1st quadrant at (2.5, 3.75) **and** y intercept at (0, 10)
(ii)		Explanation	C	2.4a	C1 for a start to explanation, e.g. $f(x+2)-3$ is a translation of $\begin{pmatrix}-2\\-3\end{pmatrix}$ or attempt to draw graph of $f(x+2)-3$ or Graph of $y=f(x+2)$ and $y=3$ drawn or $(x+2)^2-5(x+2)+10-3=0$ $(x^2-x+1=0)$
			C	2.4a	C1 for a convincing explanation, e.g. new minimum at (0.5, 0.75) so graph will not cross x axis **or** no interception between $y=f(x+2)$ and $y=3$ or demonstration that $x^2-x+1=0$ has no real roots

Q11.

Question	Working	Answer	Notes
		119	M1 for 1.06×100 oe M1 for $1.06^3\times100$ oe A1 accept 119.1016

Q12.

Paper 1MA1: 3H			
Question	**Working**	**Answer**	**Notes**
(a)	$F(x) = x^3 + 4x - 1$ $F(0) = -1, F(1) = 4$	Shown	M1 Method to establish at least one root in [0,1] eg. $x^3 + 4x - 1 (= 0)$ and F(0) (= −1), F(1) (= 4) oe A1 Since there is a sign change there must be at least one root in $0 < x < 1$ (as F is continuous)
(b)	$4x = 1 - x^3$ Or $\frac{x^3}{4} + x = \frac{1}{4}$	Shown	C1 C1 for at least one correct step and no incorrect ones
(c)	$x_1 = \frac{1}{4} - \frac{0}{4} = \frac{1}{4}$ $x_2 = \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{4} = \frac{1}{4} - \frac{1}{256}$	0.246(09375) Or $\frac{63}{256}$	B1 $x_1 = \frac{1}{4}$ M1 M1 for $x_2 = \frac{1}{4} - \frac{\left(\frac{1}{4}\right)^3}{4}$ A1 A1 for 0.246(09375) or $\frac{63}{256}$ oe

Q13.

Paper 1MA1: 2H			
Question	**Working**	**Answer**	**Notes**
		10169 or 10170	P1 for correct use of formula to find number in 2016, eg. 1.05(9500 – 250) (= 9712.5) P1 for complete iterative process, eg. 2017: 1.05(9712.5 – 250) (= 9935.625) 2018: 1.05(9935.625 – 250) C1 for answer of 10169.90... correctly rounded or truncated to nearest whole number

Q14.

Paper 1MA1:3H			
Question	**Working**	**Answer**	**Notes**
(a)		Re arrangement	M1 for re arranging to $x^3 =$ C1 a clear step to show re arrangement
(b)	$x_1 = 3.29296875$ $x_2 = 3.276659786$ $x_3 = 3.279420685$	3.28	M1 for one correct iteration M1 for 2 further iterations seen A1 cao
(c)		Statement	C1 Statement eg iteration is an estimation of the solution